Énergie mécanique et équation du mouvement

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Essai de calcul littéral pour la première équation

$$equa := -\left(1+\left(\frac{\partial}{\partial x}\,z(x)\right)^2\right)^2\cdot\mathrm{ksm}\cdot x+\left(\frac{\partial}{\partial x}\,z(x)\right)\cdot\left(g\cdot\left(1+\left(\frac{\partial}{\partial x}\,z(x)\right)^2\right)+\left(\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\,z(x)\right)\right)\cdot 2\cdot(Emsm-g\cdot z(x))\right)=0$$

$$-\left(1+\left(\frac{d}{dx}\,z(x)\right)^2\right)^2 ksm\, x+\left(\frac{d}{dx}\,z(x)\right)\left(g\left(1+\left(\frac{d}{dx}\,z(x)\right)^2\right)+2\left(\frac{d^2}{dx^2}\,z(x)\right)(Emsm-g\,z(x))\right)=0 \tag{1}$$

dsolve(equa)

$$z(x)=(\_b(\_a))\ \&\text{where}\left[\left\{\frac{1}{1+\left(\frac{d}{d\_a}\,\_b(\_a)\right)^2}\left(-\_a^2\,ksm\left(\frac{d}{d\_a}\,\_b(\_a)\right)^2-2\left(\frac{d}{d\_a}\,\_b(\_a)\right)^2\_C1\,ksm+2\,Emsm\left(\frac{d}{d\_a}\,\_b(\_a)\right)^2-ksm\,\_a^2+2\,g\,\_b(\_a)-2\,\_C1\,ksm\right)=0\right\},\{\_a=x,\_b(\_a)=z(x)\},\{x=\_a,z(x)=\_b(\_a)\}\right] \tag{2}$$

Essai avec l'intégrale première ainsi obtenue, dans le cas particulier C1=0 correspondant à Ep(0)=0

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$$equa := -\left(1+\left(\frac{\partial}{\partial x}\,z(x)\right)^2\right)\cdot\mathrm{ksm}\cdot x^2+2\cdot g\cdot z(x)+2\cdot Emsm\cdot\left(\frac{\partial}{\partial x}\,z(x)\right)^2=0$$

$$-\left(1+\left(\frac{d}{dx}\,z(x)\right)^2\right) ksm\,x^2+2\,g\,z(x)+2\,Emsm\left(\frac{d}{dx}\,z(x)\right)^2=0 \tag{3}$$

dsolve(*equa*, *z*(*x*), *series*)

$$z(x)=z(0)+RootOf\left(g\,z(0)+Emsm\,\_Z^2\right)x-\frac{1}{4}\,\frac{g}{Emsm}\,x^2+\frac{1}{12}\,\frac{ksm\,(-g\,z(0)+Emsm)}{Emsm^2\,RootOf\left(g\,z(0)+Emsm\,\_Z^2\right)}\,x^3-\frac{1}{48}\,\frac{ksm\,(2\,g\,z(0)+Emsm)}{Emsm^2\,z(0)}\,x^4+\frac{1}{160}\,\frac{ksm\left(-3\,z(0)^2 g^2\,ksm+2\,Emsm\,ksm\,g\,z(0)+Emsm^2\,ksm-Emsm\,g^2\right)}{Emsm^3\,g\,z(0)\,RootOf\left(g\,z(0)+Emsm\,\_Z^2\right)}\,x^5+\mathrm{O}(x^6) \tag{4}$$

La réponse obtenue semblant incompréhensible (coefficient infini),on reprends à la main

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$zd := \alpha \cdot x^2 + \beta \cdot x^4$

$$\beta x^4 + \alpha x^2 \qquad (5)$$

$dz := diff(zd, x)$

$$4 \beta x^3 + 2 \alpha x \qquad (6)$$

$leftpart := -(1 + dz^2) \cdot \mathrm{ksm} \cdot x^2 + 2 \cdot g \cdot zd + 2 \cdot Emsm \cdot dz^2$

$$-\left(\left(4 \beta x^3 + 2 \alpha x\right)^2 + 1\right) ksm\, x^2 + 2 g \left(\beta x^4 + \alpha x^2\right) + 2\, Emsm \left(4 \beta x^3 + 2 \alpha x\right)^2 \qquad (7)$$

$expand\left(-\left(1 + \left(2 \alpha x + 4 \beta x^3\right)^2\right) ksm\, x^2 + 2 g \left(\alpha x^2 + \beta x^4\right) + 2\, Emsm \left(2 \alpha x + 4 \beta x^3\right)^2\right)$

$$-16 \beta^2 ksm\, x^8 + 32\, Emsm\, \beta^2 x^6 - 16 \alpha \beta\, ksm\, x^6 + 32\, Emsm\, \alpha \beta x^4 - 4 \alpha^2 ksm\, x^4 + 2 \beta g x^4 + 8\, Emsm\, \alpha^2 x^2 + 2 \alpha g x^2 - ksm\, x^2 \qquad (8)$$

$sort(\%)$

$$-16 \beta^2 ksm\, x^8 + 32\, Emsm\, \beta^2 x^6 - 16 \alpha \beta\, ksm\, x^6 + 32\, Emsm\, \alpha \beta x^4 - 4 \alpha^2 ksm\, x^4 + 2 \beta g x^4 + 8\, Emsm\, \alpha^2 x^2 + 2 \alpha g x^2 - ksm\, x^2 \qquad (9)$$

#on simplifie par x2 et on considère pour x=0

$equa1 := 8\, Emsm\, \alpha^2 + 2 \alpha g - ksm = 0$

$$8\, Emsm\, \alpha^2 + 2 \alpha g - ksm = 0 \qquad (10)$$

$solve(equa1, \alpha)$

$$\frac{1}{8} \frac{-g + \sqrt{8\, Emsm\, ksm + g^2}}{Emsm}, -\frac{1}{8} \frac{g + \sqrt{8\, Emsm\, ksm + g^2}}{Emsm} \qquad (11)$$

#on simplifie encore par x2 et on considère pour x=0

$equa2 := 32\, Emsm\, \alpha \beta - 4 \alpha^2 ksm + 2 \beta g = 0$

$$32\, Emsm\, \alpha \beta - 4 \alpha^2 ksm + 2 \beta g = 0 \qquad (12)$$

$solve(equa2, \beta)$

$$\frac{2 \alpha^2 ksm}{16\, Emsm\, \alpha + g} \qquad (13)$$

$\alpha := \frac{1}{8} \frac{-g + \sqrt{g^2 + 8\, Emsm\, ksm}}{Emsm}$

$$\frac{1}{8} \frac{-g + \sqrt{8\, Emsm\, ksm + g^2}}{Emsm} \qquad (14)$$

equa2

$$4 \left(-g + \sqrt{8\, Emsm\, ksm + g^2}\right) \beta - \frac{1}{16} \frac{\left(-g + \sqrt{8\, Emsm\, ksm + g^2}\right)^2 ksm}{Emsm^2} + 2 \beta g = 0 \qquad (15)$$

$solve(equa2, \beta)$

$$\frac{1}{16} \frac{ksm \left(4\, Emsm\, ksm - g \sqrt{8\, Emsm\, ksm + g^2} + g^2\right)}{Emsm^2 \left(2 \sqrt{8\, Emsm\, ksm + g^2} - g\right)} \qquad (16)$$

Essai de calcul littéral pour la seconde équation

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$$equa := -\left(1+Z(x)^2\right)\cdot \mathrm{ksm}\cdot \mathrm{x} + Z(x)\cdot\left(\left(\frac{\partial}{\partial x}Z(x)\right)\cdot ksm\cdot\left(A^2-x^2\right)+g\right)=0$$

$$-\left(1+Z(x)^2\right) ksm\, x + Z(x)\left(\left(\frac{\mathrm{d}}{\mathrm{d}x}Z(x)\right) ksm\left(A^2-x^2\right)+g\right)=0 \quad \mathbf{(17)}$$

dsolve(*equa*, *Z*(*x*), *series*)

$$Z(x)=Z(0)-\frac{g}{ksm\,A^2}x+\frac{1}{2}\,\frac{Z(0)^2+1}{Z(0)\,A^2}x^2-\frac{1}{3}\,\frac{g\left(2\,Z(0)^2-1\right)}{Z(0)^2A^4\,ksm}x^3+\frac{1}{8}\,\frac{3\,A^2Z(0)^4ksm^2+2\,Z(0)^2ksm^2A^2-ksm^2A^2+2\,g^2}{ksm^2Z(0)^3A^6}x^4-\frac{1}{15}\,\frac{g\left(8\,A^2Z(0)^4ksm^2-3\,Z(0)^2ksm^2A^2+4\,ksm^2A^2-3\,g^2\right)}{ksm^3A^8Z(0)^4}x^5+\mathrm{O}\left(x^6\right) \quad \mathbf{(18)}$$

$$zd := \alpha\cdot x^2+\beta\cdot x^4$$

$$\beta x^4+\alpha x^2 \quad \mathbf{(19)}$$

$$dz := diff(zd, x)$$

$$4\,\beta x^3+2\,\alpha x \quad \mathbf{(20)}$$

$$ddz := diff(dz, x)$$

$$12\,\beta x^2+2\,\alpha \quad \mathbf{(21)}$$

$$leftpart := -\left(1+dz^2\right)\cdot \mathrm{ksm}\cdot x+dz\cdot\left(ddz\cdot ksm\cdot\left(A^2-x^2\right)+g\right)$$

$$-\left(\left(4\,\beta x^3+2\,\alpha x\right)^2+1\right) ksm\,x+\left(4\,\beta x^3+2\,\alpha x\right)\left(\left(12\,\beta x^2+2\,\alpha\right) ksm\left(A^2-x^2\right)+g\right) \quad \mathbf{(22)}$$

$$expand\left(-\left(1+\left(2\,\alpha x+4\,\beta x^3\right)^2\right) ksm\,x+\left(2\,\alpha x+4\,\beta x^3\right)\left(\left(2\,\alpha+12\,\beta x^2\right) ksm\left(A^2-x^2\right)+g\right)\right)$$

$$48\,A^2\beta^2 ksm\,x^5-64\,\beta^2 ksm\,x^7+32\,A^2\alpha\beta\,ksm\,x^3-48\,\alpha\beta\,ksm\,x^5+4\,A^2\alpha^2 ksm\,x-8\,\alpha^2 ksm\,x^3+4\,\beta g x^3+2\,\alpha g x-ksm\,x \quad \mathbf{(23)}$$

sort(%, *x*)

$$-64\,\beta^2 ksm\,x^7+48\,A^2\beta^2 ksm\,x^5-48\,\alpha\beta\,ksm\,x^5+32\,A^2\alpha\beta\,ksm\,x^3-8\,\alpha^2 ksm\,x^3+4\,\beta g x^3+4\,A^2\alpha^2 ksm\,x+2\,\alpha g x-ksm\,x \quad \mathbf{(24)}$$

#on simplifie par x et on considère pour x=0

$$equa1 := 4\,A^2\alpha^2 ksm+2\,\alpha g-ksm=0$$

$$4\,A^2\alpha^2 ksm+2\,\alpha g-ksm=0 \quad \mathbf{(25)}$$

solve(*equa1*, α)

$$\frac{1}{4}\,\frac{-g+\sqrt{4\,A^2ksm^2+g^2}}{ksm\,A^2},\ -\frac{1}{4}\,\frac{g+\sqrt{4\,A^2ksm^2+g^2}}{ksm\,A^2} \quad \mathbf{(26)}$$

#on simplifie encore par x2 et on considère pour x=0

$$equa2 := 32\,A^2\alpha\beta\,ksm-8\,\alpha^2 ksm+4\,\beta g=0$$

$$32\,A^2\,\alpha\,\beta\,ksm - 8\,\alpha^2\,ksm + 4\,\beta\,g = 0 \tag{27}$$

$solve(equa2, \beta)$

$$\frac{2\,\alpha^2\,ksm}{8\,A^2\,\alpha\,ksm + g} \tag{28}$$