Calcul de fonctions implicites par développement limité

$f := \ln(x \cdot y) + y^{\frac{3}{2}} - 1$

$$\ln(x\,y) + y^{3/2} - 1 \tag{1}$$

$x := 1 + \varepsilon$

$$1 + \varepsilon \tag{2}$$

$y := 1 + \lambda \cdot \varepsilon + \mu \cdot \varepsilon^2 + \nu \cdot \varepsilon^3 + \rho \cdot \varepsilon^4$

$$1 + \lambda\,\varepsilon + \mu\,\varepsilon^2 + \nu\,\varepsilon^3 + \rho\,\varepsilon^4 \tag{3}$$

$\mathrm{series}(f, \varepsilon = 0, 5)$

$$\left(\frac{5}{2}\lambda + 1\right)\varepsilon + \left(\frac{5}{2}\mu + \lambda + \frac{1}{2}(-\lambda - 1)(\lambda + 1) + \frac{3}{8}\lambda^2\right)\varepsilon^2 + \left(\frac{5}{2}\nu + \frac{3}{4}\lambda\mu - \frac{1}{16}\lambda^3 + \mu + \frac{1}{3}(-\lambda - 1)(\mu + \lambda) + \frac{1}{3}(-2\mu + \lambda^2 + 1)(\lambda + 1)\right)\varepsilon^3 + \left(\frac{5}{2}\rho + \frac{3}{4}\lambda\nu + \frac{3}{8}\mu^2 - \frac{3}{16}\lambda^2\mu + \frac{3}{128}\lambda^4 + \nu + \frac{1}{4}(-\lambda - 1)(\nu + \mu) + \frac{1}{4}(-2\mu + \lambda^2 + 1)(\mu + \lambda) + \frac{1}{4}(-3\nu + 3\lambda\mu - \lambda^3 - 1)(\lambda + 1)\right)\varepsilon^4 + O(\varepsilon^5) \tag{4}$$

$solve\left(\frac{5}{2}\lambda + 1 = 0, \lambda\right)$

$$-\frac{2}{5} \tag{5}$$

$subs\left(\lambda = -\frac{2}{5}, \frac{5}{2}\mu + \lambda + \frac{1}{2}(-\lambda - 1)(\lambda + 1) + \frac{3}{8}\lambda^2\right)$

$$\frac{5}{2}\mu - \frac{13}{25} \tag{6}$$

$solve\left(\frac{5}{2}\mu - \frac{13}{25} = 0, \mu\right)$

$$\frac{26}{125} \tag{7}$$

$subs\left(\lambda = -\frac{2}{5}, \mu = \frac{26}{125}, \frac{5}{2}\nu + \frac{3}{4}\lambda\mu - \frac{1}{16}\lambda^3 + \mu + \frac{1}{3}(-\lambda - 1)(\mu + \lambda) + \frac{1}{3}(-2\mu + \lambda^2 + 1)(\lambda + 1)\right)$

$$\frac{5}{2}\nu + \frac{421}{1250} \tag{8}$$

$solve\left(\frac{5}{2}\nu + \frac{421}{1250} = 0, \nu\right)$

$$-\frac{421}{3125} \tag{9}$$

$$subs\left(\lambda = -\frac{2}{5}, \mu = \frac{26}{125}, \nu = -\frac{421}{3125}, \frac{5}{2}\rho + \frac{3}{4}\lambda\nu + \frac{3}{8}\mu^2 - \frac{3}{16}\lambda^2\mu + \frac{3}{128}\lambda^4 + \nu + \frac{1}{4}(-\lambda - 1)(\nu + \mu) + \frac{1}{4}\left(-2\mu + \lambda^2 + 1\right)(\mu + \lambda) + \frac{1}{4}\left(-3\nu + 3\lambda\mu - \lambda^3 - 1\right)(\lambda + 1)\right)$$

$$\frac{5}{2}\rho - \frac{6191}{25000} \tag{10}$$

$$solve\left(\frac{5}{2}\rho - \frac{6191}{25000} = 0, \rho\right)$$

$$\frac{6191}{62500} \tag{11}$$

restart