Repulsive gravitation and “black holes”

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Abstract

In general relativity, within the study of the central symmetric gravitational field, an attentive care in the physical interpretation of the mathematical solutions solves some difficulties related to the interpretation of the coordinates and highlights situations where the gravitation is repulsive. From it is deduced the impossibility of existence of stellar “black holes” in the simplistic sense of this expression. A simplified, but probably qualitatively effective, modelling allows to suggest an interpretation of the nature of the galactic “black holes” candidates, maybe even a reinterpretation of the big-bang.

1. Introduction

The ten equations of the gravitational field [1, 2] $(R^{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}R = \chi\, T^{\alpha\beta})$ are not independent. This leaves a certain freedom of choice of the coordinates.

For a star with spherical symmetry, the most general form can be written:

$$ds^2 = A(r, t)\, c^2\, dt^2 + B(r, t)\, c\, dt\, dr - C(r, t)\, dr^2 - D(r, t)\, [d\theta^2 + \sin^2(\theta)\, d\varphi^2].$$

In the static case, one can use a metric of the following form:

$$ds^2 = A(r)\, c^2\, dt^2 - C(r)\, dr^2 - D(r)\, [d\theta^2 + \sin^2(\theta)\, d\varphi^2].$$

The gravitational information is actually a priori contained in the expressions D(r) concerning the geometry of space and A(r) for what relates to time; the expression C(r) doing nothing but depend on the choice of the radial variable used.

One can choose the “classic” notations for which $D(r) = r^2$; this consists in considering that, by observing the star from the outside, we perceive it by its perimeter, here equal to $2\pi r$ as in Euclidian geometry.

However, while thus proceeding, one defers the geometrical information of D(r) in the way in which the coordinate r varies, according to the position in space. This is not prohibited, but it is then necessary to carefully study the physical interpretation of the metric. In particular it may be prudent to check that one finds the same results with other notations (what provides relativistic invariance).

One can choose the “isotropic” coordinates, for which $D(r) = C(r)\, r^2$. One will note in this case: $ds^2 = A(\underline{r})\, c^2\, dt^2 - \underline{C}(\underline{r})\, \{d\underline{r}^2 - \underline{r}^2\, [d\theta^2 + \sin^2(\theta)\, d\varphi^2]\}$.

One can choose the “radial” coordinates, for which $C(r) = 1$; this consists in choosing as radial variable the distance to the centre of the star (not measurable inside). The calculations are generally not simple with this variable, which will be noted ρ, but it can be used for some comparisons.

2. Study of the external case

2.1. Highlighting the ambiguity of some notations

With the “classic” notations, the resolution of the field equations gives: $C = \frac{1}{A}$ then $A = 1 - \frac{r_s}{r}$ with $r_s = \frac{2\mathcal{G}M}{c^2}$. An important characteristic is the existence of the singularity for $r = r_s$, such as for $r < r_s$ the variable t ceases being of the temporal type.

Many artifices have been proposed to by-pass this difficulty [1, 2] (for example coordinates of Finkelstein, or Kruskal), but a detailed study shows that it is in fact useless.

With “isotropic” notations, one obtains: $\underline{C}(\underline{r}) = \left(1 + \frac{\underline{r}_s}{\underline{r}}\right)^4$ and $A(\underline{r}) = \frac{(\underline{r} - \underline{r}_s)^2}{(\underline{r} + \underline{r}_s)^2}$,

with $\underline{r}_s = \frac{r_s}{4}$. In this case, the variable t remains of the temporal type; this different behaviour is incompatible with relativistic invariance, which suggests a bad interpretation.

The change of notation corresponds to: $r = \underline{r} \,.\left(1+\frac{\underline{r}_s}{\underline{r}}\right)^2$, but conversely: $\underline{r} = r \,.\, \frac{1}{2}\left(r-\frac{r_s}{2} \pm \sqrt{r.\left(r-r_s\right)}\right)$.

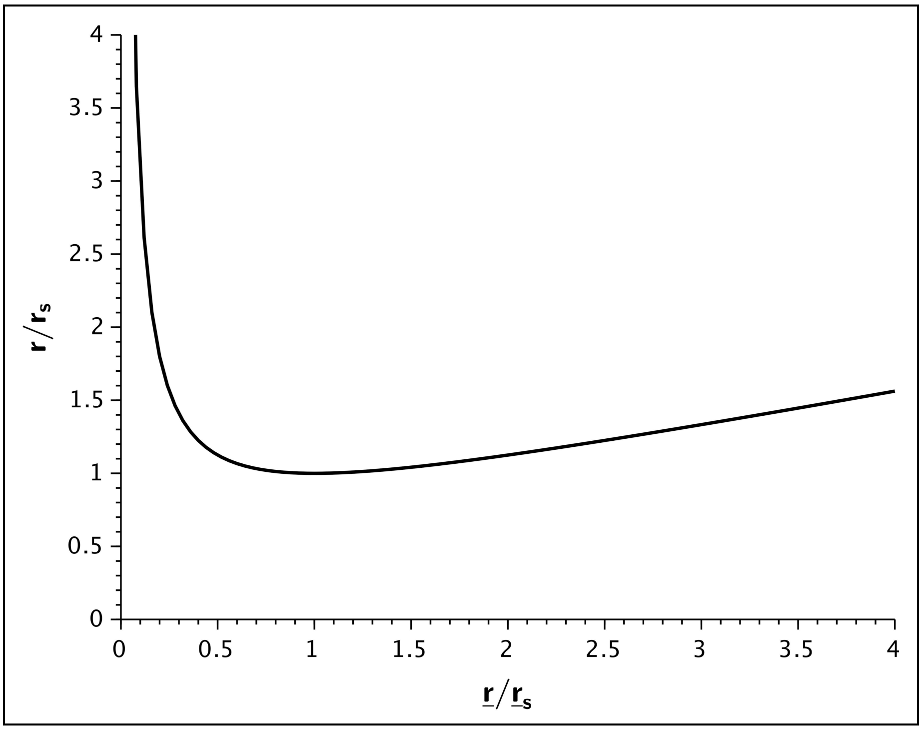


Fig. 1: variations (in reduced notations) of the “classic” radial variable as a function of the “isotropic” radial variable.

This is not bijective (fig. 1), thus a priori impossible, unless a careful re-interpretation.

2.2. Radial distances

One can compare the radial "isotropic" coordinate and the radial distance ρ:

$$\rho = \underline{r}_s \ln\left(\frac{\underline{r}^2}{\underline{r}_s^2}\right) + \underline{r}\,.\left(1-\frac{\underline{r}_s^2}{\underline{r}^2}\right) + \text{Cste.}$$

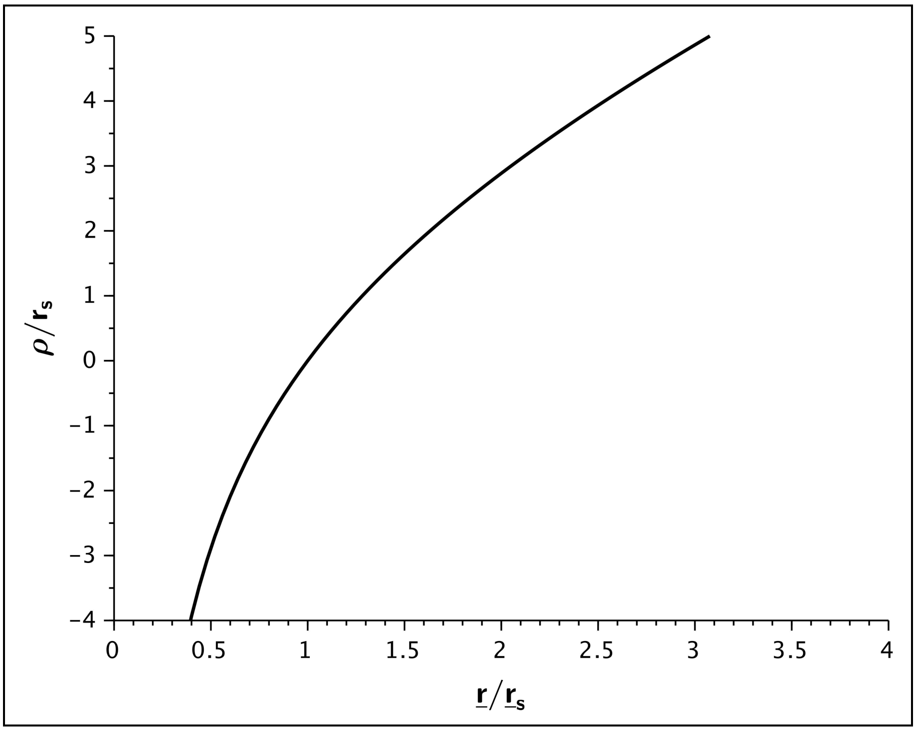


Fig. 2: variations (in reduced notations) of the radial distance as a function of the "isotropic" radial variable.

This shows occasionally (fig. 2) that the coordinate $\underline{r}$ cannot be reasonably used until $\underline{r} = 0$, but this necessarily involves the study inside the star.

One obtains for the "classic" coordinate r (considering its not monotonous sense of variation): $\rho = \pm\left(r_s \operatorname{arg\,th}\left(\sqrt{1-\frac{r_s}{r}}\right) + r\sqrt{1-\frac{r_s}{r}}\right) + \text{Cste.}$

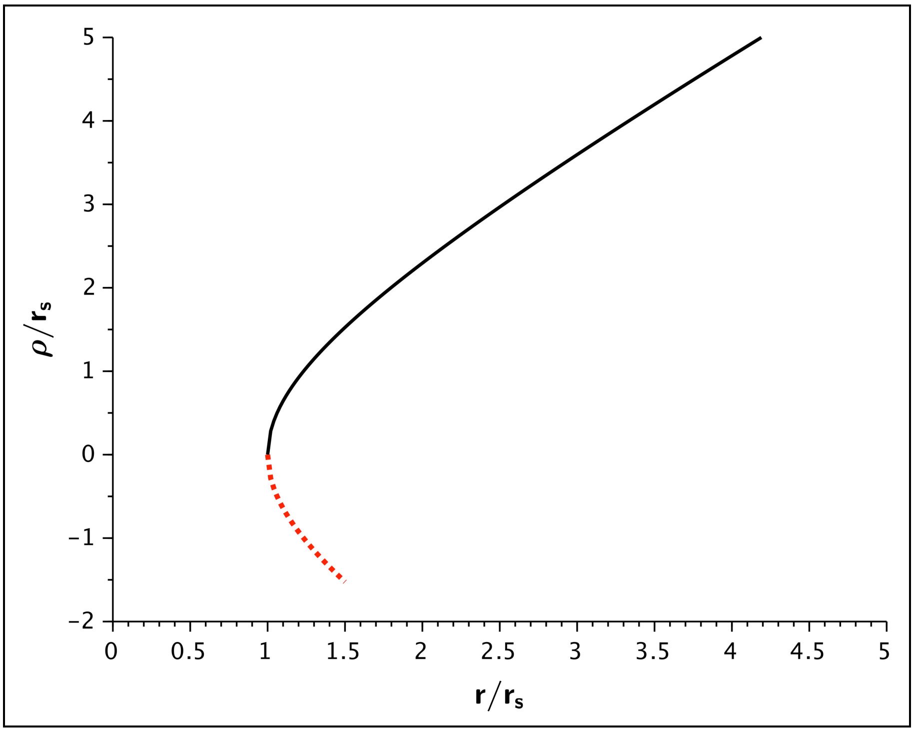


Fig. 3: variations (in reduced notations) of the radial distance as a function of the “classic” radial variable.

Even if one omits on the graph the constant of integration occurring in this relation (according to the inside of the star), it begins to appear (fig. 3) that the study of the singularity by considering $r < r_S$ is inappropriate: when one approaches the centre of the star beyond $r = r_S$, the coordinate r increases (what makes more delicate the interpretation of this variable).

2.3. Radial freefall

In “isotropic” notations, the metrics limited to the radial movement can be written: $ds^2 = A(\underline{r})\, c^2\, dt^2 - C(\underline{r})\, d\underline{r}^2$. From this are deduced the equations of geodesic motion:

$$c\frac{d^2t}{ds^2} + \frac{A'}{A}\, c\frac{dt}{ds}\frac{d\underline{r}}{ds} = 0 \quad ; \quad \frac{d^2\underline{r}}{ds^2} + \frac{A'}{2C}\, c^2\left(\frac{dt}{ds}\right)^2 + \frac{C'}{2C}\left(\frac{d\underline{r}}{ds}\right)^2 = 0.$$

The integration gives: $A\, c\frac{dt}{ds} = \sqrt{A_0}$; $\frac{d\underline{r}}{ds} = \frac{1}{\sqrt{C}}\sqrt{\frac{A_0}{A} - 1}$ (with $A_0 = A(\underline{r}_0)$).

The local duration is: $dt_\ell = \sqrt{A}\, dt$; the distance is: $d\ell = \sqrt{C}\, d\underline{r}$; the speed is therefore: $v = \frac{d\ell}{dt_\ell} = \sqrt{\frac{C}{A}}\frac{d\underline{r}}{dt} = c\sqrt{1 - \frac{A}{A_0}}$.

It appears that the falling speed tends towards c when $\underline{r} \to \underline{r}_s$. If one then assume that the point continues to move beyond this limit, it appears that $A(\underline{r}) = \left(\frac{\underline{r} - \underline{r}_s}{\underline{r} + \underline{r}_s}\right)^2$ cancels and then becomes positive again: as a result of what should be the attraction of the star, the speed decreases! One obtains the same result in classic coordinates, with $A(r) = \frac{r - r_s}{r}$ monotonous function, but with (and only with) r increasing when the point is approaching closer to the star than the singularity. This describes a repulsive gravitational effect.

Since the gravitational effects must vary continuously, one can deduce from this that, for any static star, the singularity is necessarily interior. Otherwise, the matter on the surface would undergo a repulsive gravitational field and would be expelled. Thus, at least in the static case described by general relativity, the existence of stellar “black holes” in the simplistic sense seems impossible.

However, since one observes stars appearing as “black holes”, a study of the interior case can be useful to try to understand their nature.

3. Study of the static case including inside

3.1. Model of the "simple" fluid

The field equations can be written $R^{\alpha\beta} = \chi.(T^{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}T)$.

For a "simple" fluid: $T^{\alpha\beta} = (p + \varepsilon_0)\, u^\alpha u^\beta - p\, g^{\alpha\beta}$, with $u^\alpha = \dfrac{dx^\alpha}{ds}$ and where p and $\varepsilon_0 = \mu_0\, c^2$ designate the pressure and the energy density measured in the self reference of the fluid, in this case at rest (specific mass $\mu = \mu_0$).

With a metric of the form: $ds^2 = A(r)\, c^2\, dt^2 - C(r)\, dr^2 - D(r)\, d\Omega^2$, one obtains: $u^0 = \dfrac{1}{\sqrt{A}}$ and $u^k = 0$.

This corresponds to: $T_{00} - \frac{1}{2}g_{00}T = A\,\dfrac{\varepsilon + 3p}{2}$; $T_{11} - \frac{1}{2}g_{11}T = C\,\dfrac{\varepsilon - p}{2}$;

$$T_{22} - \frac{1}{2}g_{22}T = D\,\frac{\varepsilon - p}{2} \;;\; T_{33} - \frac{1}{2}g_{33}T = D\,\sin^2(\theta)\,\frac{\varepsilon - p}{2}.$$

Moreover: $R_{00} = \dfrac{A''}{2C} - \dfrac{A'}{2C}.\left(\dfrac{A'}{2A} + \dfrac{C'}{2C} - \dfrac{D'}{D}\right)$;

$$R_{11} = -\frac{A''}{2A} - \frac{D''}{D} + \frac{A'}{2A}.\left(\frac{A'}{2A} + \frac{C'}{2C}\right) + \frac{D'}{D}.\left(\frac{C'}{2C} + \frac{D'}{2D}\right) \;;$$

$$R_{22} = -\frac{D''}{2C} - \frac{D'}{2C}.\left(\frac{A'}{2A} - \frac{C'}{2C}\right) + 1 \;;\; R_{33} = R_{22}\,\sin^2(\theta).$$

One thus obtains two combinations allowing to simplify:

$$\frac{D'}{2CD}.\left(\frac{A'}{A} + \frac{D'}{2D}\right) - \frac{1}{D} = \chi\, p \;;\; -\frac{D''}{CD} + \frac{D'}{2CD}.\left(\frac{C'}{C} + \frac{D'}{2D}\right) + \frac{1}{D} = \chi\, \varepsilon.$$

Taking into account the pressure $p = p(r)$ requires an additional equation. The conservation of the energy-momentum can be written: $D_\beta T^{\alpha\beta} = 0$; one deduces from this the law of the statics of fluids: $p' = -(\varepsilon + p)\,\dfrac{A'}{2A}$.

3.2. Global expression of the metric

One can use here the "classic" coordinates since it was shown how to use them (the "isotropic" notations gives the same results).

In the particular case $D = r^2$ one obtains $D' = 2r$ and $D'' = 2$; thus:

$$\frac{1}{Cr}.\left(\frac{A'}{A}+\frac{1}{r}\right) - \frac{1}{r^2} = \chi\, p \;\; ; \quad \frac{1}{C} - \frac{r\,C'}{C^2} = 1 - \chi\, \varepsilon\, r^2.$$

The second equation leads to: $\dfrac{r}{C} = r - \chi\, c^2 \displaystyle\int_0^r \mu(r')\, r'^2\, dr'$. One can define: $M(r) = \displaystyle\int_0^r \mu(r')\, 4\pi r'^2\, dr'$ and $a(r) = \dfrac{\chi c^2}{4\pi} M(r) = \dfrac{2\mathcal{G}}{c^2} M(r)$. The previous relation can then be written as: $C(r) = \dfrac{r}{r - a(r)}$.

The carry forward in the first equation gives, with the law of statics:

$$p' = -\frac{\varepsilon + p}{2}\,\frac{a(r) + \chi r^3 p}{r.(r - a(r))}.$$

Thus, the knowledge of $\mu(r)$ allows to calculate $a(r)$ and $C(r)$ by integration from $a(0) = 0$, then $p(r)$ by integration from $p(R) = 0$, then $A(r)$ by integration according to the law of statics: $\dfrac{A'}{A} = -\dfrac{2p'}{\varepsilon + p}$.

For a compressible fluid, the specific mass depends however on the pressure (and on the temperature). It is then necessary to suppose a law of compressibility (relation between μ and p) and to integrate the system of the two equations (then one deduces A from this).

Outside the star: $M = M(R) = \displaystyle\int_0^R \mu(r')\, 4\pi r'^2\, dr'$ and $r_s = a(R) = \dfrac{2\mathcal{G}M}{c^2}$; this gives again: $C = \dfrac{r}{r - r_s}$ and $A(r) = \dfrac{1}{C(r)}$ since $A = 1$ to infinity.

3.3. Spatial geometry for an uniform specific mass

The literal resolution is not simple in the general interior case. One can proceed by numerical integration, but one can also be interested to look for simplifying hypotheses allowing to highlight simply the dominating physical properties.

With a uniform specific mass (what can be a good qualitative approximation): $a(r) = \lambda r^3$ with $\lambda = \frac{\chi \mu c^2}{3}$.

Thus $C = \frac{r}{r - a(r)} = \frac{1}{1 - \lambda r^2}$; which requires $r < \frac{1}{\sqrt{\lambda}}$.

Since the radial distance can be written $d\rho = \pm\sqrt{C(r)}\,dr$, the "inner radius" (distance to the centre) here corresponds to: $\rho = \frac{1}{\sqrt{\lambda}} \arcsin\left(r\sqrt{\lambda}\right)$.

Conversely: $r = \frac{1}{\sqrt{\lambda}} \sin\left(\rho\sqrt{\lambda}\right)$; the boundary condition $r < \frac{1}{\sqrt{\lambda}}$ thus corresponds to $\rho < \frac{\pi}{2\sqrt{\lambda}}$; one can verify that the sizes of "known" stars agree this condition.

But it is mainly important to understand that beyond $r(\rho)$ is decreasing (if such a solution can exist physically): the limit can be reached, but never exceeded. Thus $r \geq a(r)$ everywhere inside, just as $r \geq r_s$ everywhere outside.

It seems however at this level that no physical constraint does impose any maximum size (for a given specific mass). Can there exist stars of size and mass such as the limit would be exceeded?

It is then useful to consider the variation of r according to ρ. For λ fixed (graphical representations are made taking $\frac{1}{\sqrt{\lambda}}$ as unit of length), the radii $R < \frac{1}{\sqrt{\lambda}}$ give a simple connexion (fig. 4); the singularity is never reached: $a(r) < r$ everywhere.

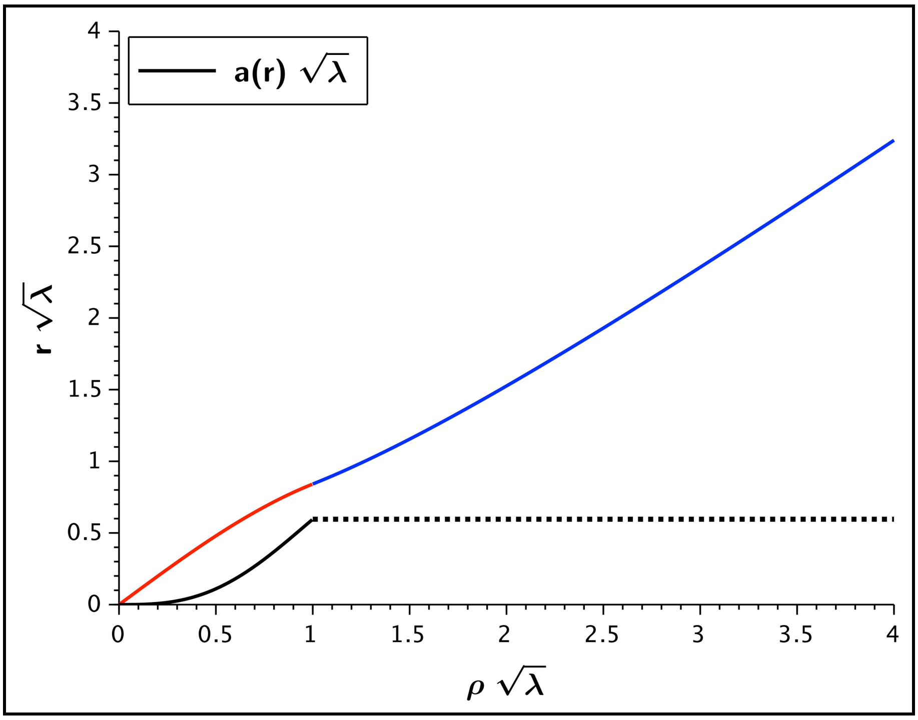


Fig. 4: variations (in reduced notations) of the "classic" radial variable as a function of the radial distance, in the monotonic case; the variations of the Schwarzschild radius a(r) associated with the "interior mass" are also shown for comparison.

The "inner radii" $\rho > \frac{\pi}{2\sqrt{\lambda}}$ correspond to $R < \frac{1}{\sqrt{\lambda}}$; they give "in principle" a supra-limit connection (fig. 5), where the singularity $r = a(r)$ is reached inside, but never exceeded: $a(r) \leq r$ and $r_s < R$, since a(r) decreases near the surface; moreover the singularity $r = r_s$ is also reached outside, since r also decreases thereafter, but $r_s \leq r$. This case requires however a more detailed study.

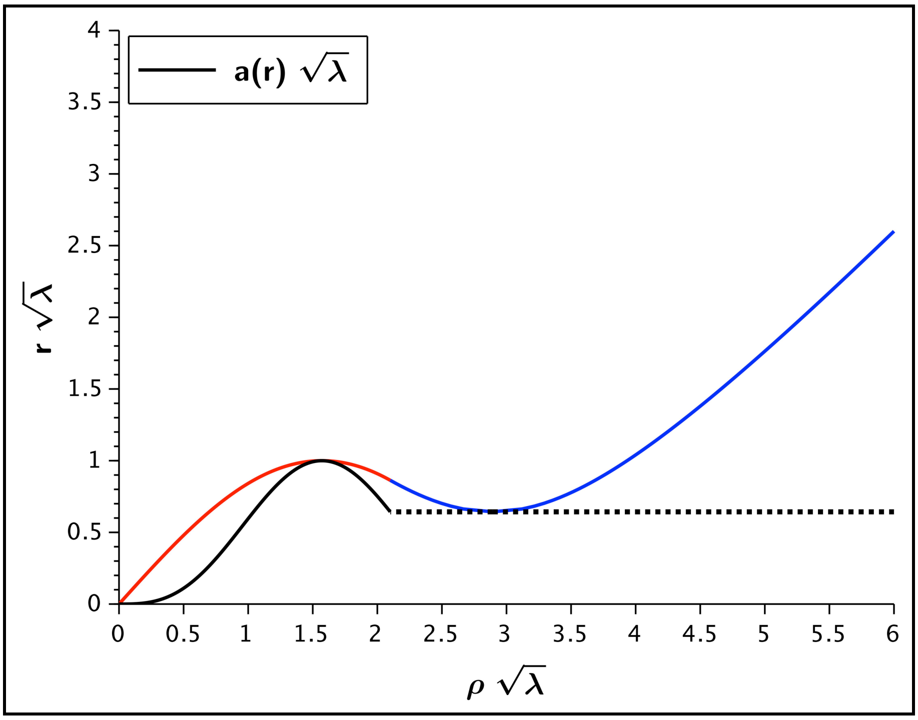


Fig. 5: variations (in reduced notations) of the "classic" radial variable as a function of the radial distance, in the case with inversion; the variations of the Schwarzschild radius a(r) associated with the "interior mass" are also shown for comparison.

3.4. "Gravitational field" for an uniform specific mass

With a uniform specific mass, one obtains furthermore:

$$\frac{2\chi\, p'}{(\lambda+\chi\, p)(3\lambda+\chi\, p)} = -\frac{\mathrm{r}}{1-\lambda \mathrm{r}^2}.$$

For a star in mechanical equilibrium, the limit $p(\mathrm{R}) = 0$ leads to:

$$p = \frac{3\lambda}{\chi}\left(\frac{\sqrt{1-\lambda \mathrm{r}^2}-\sqrt{1-\lambda \mathrm{R}^2}}{3\sqrt{1-\lambda \mathrm{R}^2}-\sqrt{1-\lambda \mathrm{r}^2}}\right).$$

One obtains then $\frac{A'}{A} = -\frac{2p'}{\varepsilon + p} = \frac{2\lambda r}{\sqrt{1-\lambda r^2}\left(3\sqrt{1-\lambda R^2} - \sqrt{1-\lambda r^2}\right)}$; taking into account the limit: $A(R) = 1 - \lambda R^2$, integration thus gives:

$$A = \frac{1}{4}.\left(3\sqrt{1-\lambda R^2} - \sqrt{1-\lambda r^2}\right)^2.$$

In the simple cases, with $\frac{dr(\rho)}{d\rho} > 0$ everywhere, that is to say $\rho(R) < \frac{\pi}{2\sqrt{\lambda}}$ and thus $r \le R < \frac{1}{\sqrt{\lambda}}$, the numerator of the expression of p remains strictly positive everywhere.

It appears on the other hand that the pressure cannot always be defined in the central zone: the expression diverges for $r \le r_d = \sqrt{9R^2 - \frac{8}{\lambda}}$.

This limit r_d below which the pressure can not be defined also corresponds to an inversion of the direction of gravity (null minimum of A, negative divergence of ln(A), which behaves like a potential).

One can avoid the divergence by imposing $R < \sqrt{\frac{8}{9}}\,\frac{1}{\sqrt{\lambda}} \approx \frac{0{,}94}{\sqrt{\lambda}}$ (thus $r_d < 0$), but it is then necessary to justify this constraint by a physical reasoning.

This limit means the impossibility of equilibrium (a priori assumed in the study of the static case).

One can moreover show [2] that it is a relatively “fundamental” physical limit, not restricted to the case of uniform mass density. If this limit is exceeded, the pressure diverges necessarily at the centre of the star, which requires a study within the ultra-relativist limit.

3.5. Ultra-relativist limit

Taking into account the relation $\frac{da(r)}{dr} = \chi\, r^2\, \varepsilon$ the ultra-relativist case $p = \frac{\varepsilon}{3}$ allows to write the equation: $p' = -\frac{\varepsilon + p}{2}\,\frac{a(r) + \chi r^3 p}{r.(r - a(r))}$ in the form:

$$3\,a''\,r.(r - a) = 2\,a'.(3r - 6a - r\,a').$$

The only physically acceptable solution respecting the limit $a(0) = 0$ is: $a(r) = \frac{3}{7}\,r$. That corresponds to an energy density $\varepsilon(r) = \frac{3}{7\chi r^2}$ divergent at the origin (but integrable). The divergence of $\varepsilon(r)$ at the origin means that the temperature tends towards infinite at the origin (and the pressure also).

On the other hand, no star can be entirely ultra-relativist since the pressure thus calculated vanishes only to the infinite. To describe the transition towards the ultra-relativist behaviour, it is necessary to add an equation of state connecting ε and p.

Interesting results can be obtained with a polytropic model (with $p \propto \varepsilon^{4/3}$; [2] part III.11.5). It is however useful to note that a rudimentary modelling can be sufficient to find a quasi-equivalent qualitative description. One may thus propose a simple mathematical transition: $\varepsilon(r) = 3\,p + \frac{\varepsilon_R^2}{\varepsilon_R + p}$, with $\varepsilon_R = \varepsilon(R)$.

With $\frac{da(r)}{dr} = \chi\, r^2\, \varepsilon(r)$ and $\lambda = \frac{\chi\,\varepsilon_R}{3}$, the carry-forward in the relation of equilibrium leads to the equation:

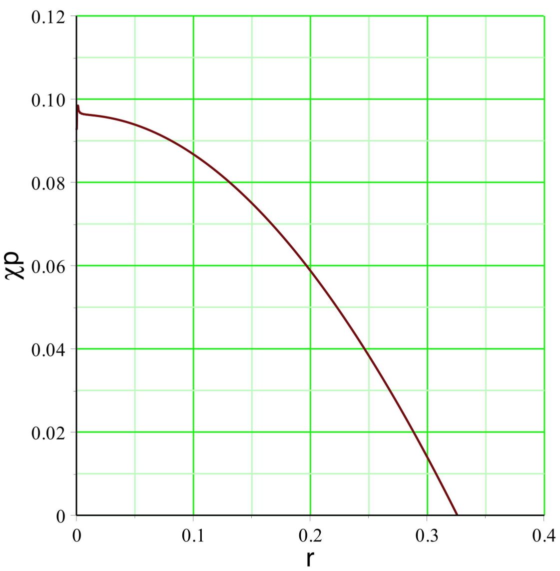
$$12\,(a''\,r - 2\,a').(r - a).\left[a' + 9\lambda r^2 + \sqrt{a'^2 + 18a'\lambda r^2 - 27\lambda^2 r^4}\right] =$$

$$= -\sqrt{a'^2 + 18a'\lambda r^2 - 27\lambda^2 r^4}\,.\left[7a' - 9\lambda r^2 + \sqrt{a'^2 + 18a'\lambda r^2 - 27\lambda^2 r^4}\right] \times$$

$$\times \left(6a + r.\left[a' - 9\lambda r^2 + \sqrt{a'^2 + 18a'\lambda r^2 - 27\lambda^2 r^4}\right]\right).$$

To simplify the notations, one can take $\frac{1}{\sqrt{\lambda}}$ as unit of length (this numerically corresponds to use $\lambda = 1$).

There is no simple literal solution; numerical integration gives solutions whose form well describes the transition from that obtained with a low uniform mass density (fig. 6.a; the effect of the pressure here is very limited) to that obtained for the ultra-relativist limit (fig. 6.b; the pressure diverges in the centre, but vanishes on the surface of the star).



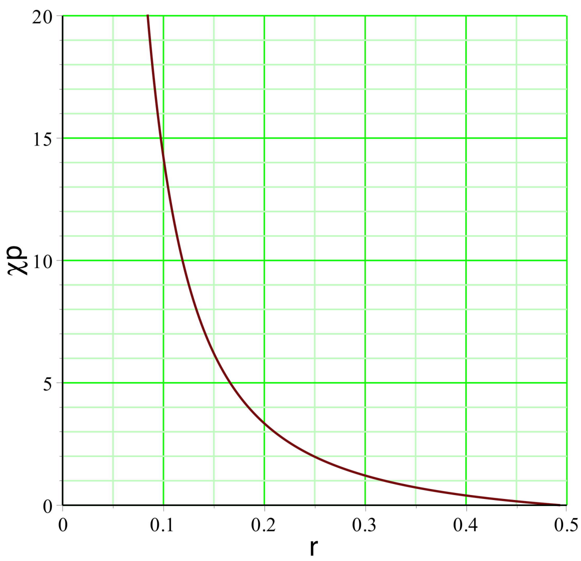


Fig. 6.a and b: variations (in reduced notations) of the pressure as a function of the “classic” radial variable, in a non-relativistic case (a); in the ultra-relativistic case (b).

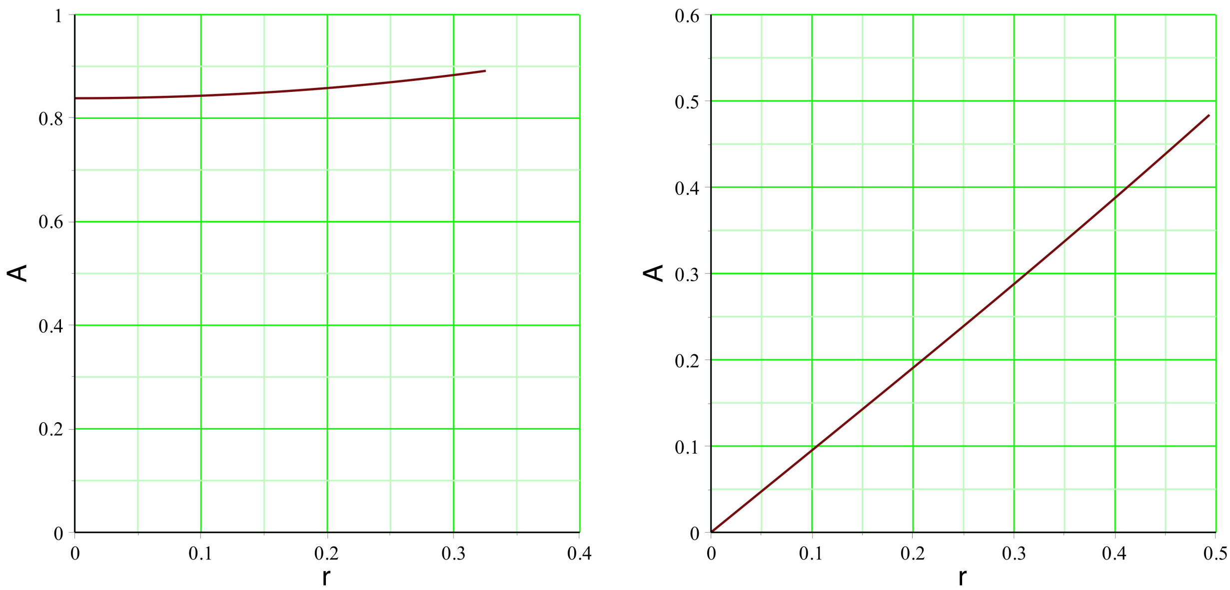
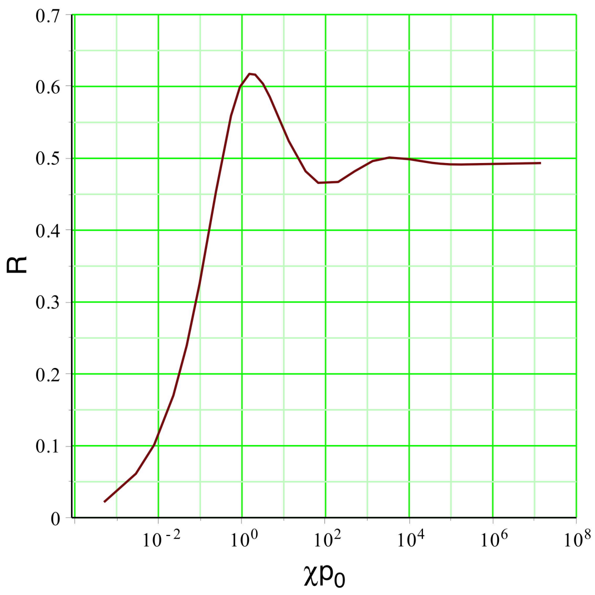


Fig. 7.a and b: variations (in reduced notations) of coefficient A of the metric as a function of the “classic” radial variable, in a non-relativistic case (a); in the ultra-relativistic case (b).

From the point of view of metric, the ultra-relativistic limit is characterized by a coefficient A tending towards zero to the centre (fig. 7.a and 7.b). Thus, since ln(A) behaves like a potential of gravitation, the field tends towards infinity to the centre (just as ε and p).

For “small” stars (low pressure in the centre) one finds rather logically that the radius R (value of r for which the pressure vanishes) increases as a function of the central pressure p_0 (and reciprocally).

On the other hand (fig. 8.a), for the stars whose central pressure is larger, the radius R does not increase any more (and even transitorily decreases a little). This property, which can surprise, comes from the fact that the increase in pressure causes an increase in the “gravitational field”, which changes the metric.



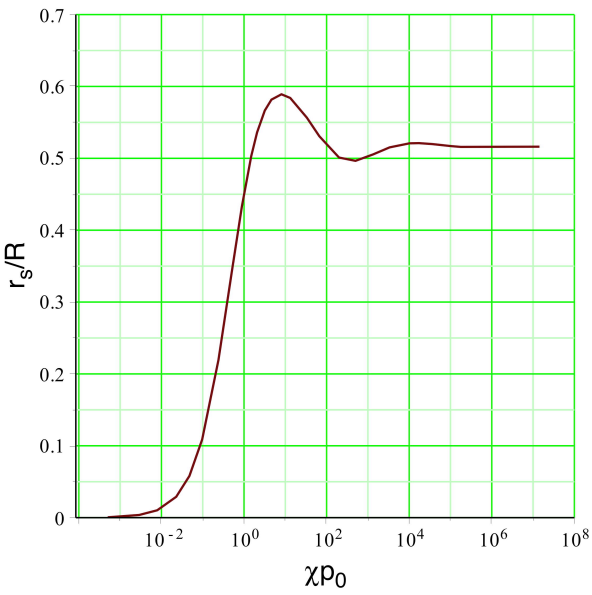


Fig. 8.a and b: variations (in reduced notations) of the “classic” radius R of the star as a function of the central pressure (a); relative variations or the Schwarzschild radius r_s/R (b).

The evolution of $\frac{r_s}{R}$ shows in the same way (fig. 8.b) a limit $\approx 0{,}55 < 1$; this means that the limit $r_s = a(R) = R$ corresponding to the formation of black holes is never reached.

During numerical integration, the adjustment of the constants in order to observe the boundary conditions suggests that this characteristic behaviour is only due to the approach of the ultra-relativist limit. This concerns an “unstable” solution: even if one imposes important modifications on the centre, the solution converges towards a limit practically identical to the periphery. Conversely, weak disturbances on the surface can cause a notable destabilization of the centre of the star. Thus, any star whose central part would become ultra-relativist would be unstable.

3.6. Star exceeding the limit

A question then arises inevitably: what happens if, on the occasion of the cosmic movements, an accumulation of matter exceeds at a given moment the preceding limit?

The infinite pressure suggests the occurrence of a shock wave, as violent as those of the supernova, but all depends in fact on how the limit is exceeded.

In some cases, the limit is exceeded “softly”: the matter coming from a binary star companion falls gradually in increasingly tightened spiral and forms an accretion disc.

This matter reaches the ground at a speed close to that of the light, but this matter cannot accumulate because the star becomes unstable: under the effect of the shock wave, matter is violently ejected along the polar axis.

Part of the energy of the incident matter is dissipated by frictions during the shock on the surface, but that tends to raise the temperature of the star, therefore also its (relativistic) mass density, which decreases the limit in size. Thus some matter is ejected in an amount at least equal, even higher, to the incidental quantity: the size of the star rather tends to decrease, not by compactification, but by matter ejection. The star always remains below the limit that would form a “black hole”.

The situation is different if two stars, already close to the limit, become to collide. The violence of the shock depends on its geometry: frontal shock or progressive approach in spiral, but always leads to a supernova. A restricted part of the matter remains grouped and forms a new star; the remainder is ejected according to a geometry (spherical or axial) which depends on that of the shock.

While it is clear that the violence of the shock is sufficient to explain an explosion, another fundamental effect occurs: during a relatively short time, a matter cluster higher than the limit is formed, which implies an inversion of the direction of variation of $r(\rho)$.

In the cases with “inversion”, that is to say with $\frac{dr(\rho)}{d\rho} < 0$ in an intermediate zone, there is always $r \leq \frac{1}{\sqrt{\lambda}}$ and $R < \frac{1}{\sqrt{\lambda}}$, but $r > R$ in some places. The numerator of the expression of p obtained previously then does not remain everywhere positive; this inconsistency means in fact as well an impossibility of equilibrium (even if $R < \sqrt{\frac{8}{9}} \frac{1}{\sqrt{\lambda}}$).

But on another side, even if it is not obvious to solve the complete system of equations under these conditions, general considerations allow to deduce from it the physical properties starting from a partial solution.

By comparison with the Newtonian case, the relativistic equation of statics expresses that $\ln\left(\sqrt{A}\right)$ plays a part similar to that of the potential of gravitation:

$$\partial_i p = -(p + \varepsilon_0) \frac{\partial_i A}{2A} = -(p + \varepsilon_0)\, \partial_i\left(\ln\left(\sqrt{A}\right)\right).$$

in the area outside the star, one can calculate $A(r(\rho)) = \frac{1}{C(r(\rho))}$; moreover, continuity indicates how behaves A(r) inside, in the vicinity of surface.

The decrease of the potential, in the zone with inversion, describes a repulsive field of gravitation [3]. This property is inevitably prolonged in the interior part close to surface (fig. 9): the matter of surface is ejected.

In this case, even more than in that without “inversion”, the phenomenon leads to a shock wave and a supernova. Thus, as long as one supposes valid the general relativity, it is not easily conceivable that a stellar “black hole” can remain long enough to be observed.

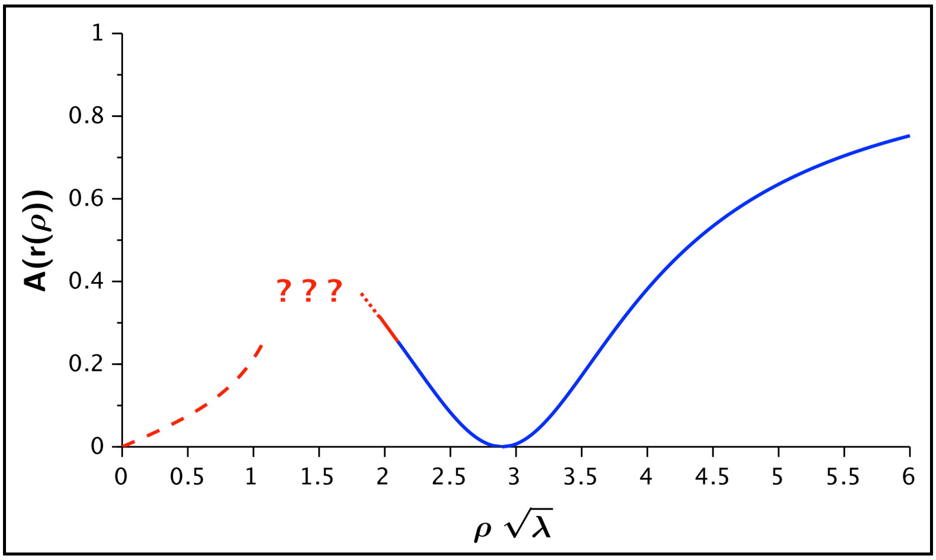


Fig. 9: variations (in reduced notations) of coefficient A of the metric as a function of the radial distance (case with inversion).

Consequently, if under the conditions without inversion where the central pressure tends towards infinite, one can imagine that this causes a collapse in a form denser than neutron stars (star with plasma of quarks, approximately ten times denser, currently only hypothetical), it is clear that this cannot form a black hole, but can at most cause an inversion, thus in any case can only finish in supernova.

Taking this into account, what can be thought about the supposed black holes observed indirectly by their interaction with a companion star? They are most probably dark dwarf: neutron stars at the end of their life, whose mass is very close to the limit of stability and whose behaviour, observed by far, is nearly identical to that of a black hole.

In fact, so that the light passing in the vicinity of such dark dwarf gives it such an appearance, it is not necessary for its radius R to be “interior” to the singularity at $r = r_s$ (what would correspond to a black hole, but with $R > r_s$ because of the inversion), it is sufficient that $r_s < R < \sqrt{\frac{27}{4}}\, r_s \approx 2{,}6\, r_s$.

4. Galactic "black hole"

Conversely, one can consider a "black hole" candidate in the centre of a galaxy, assuming it almost near the limit of stability $(R \approx \frac{1}{\sqrt{\lambda}})$. With a mass $M = 2{,}6.10^6\ M_S$, one obtains: $R \approx 8.10^6$ km and $\mu \approx 2{,}6.10^6\ kg.m^{-3}$.

This specific mass is very far from the maximum concentration allowed by the corpuscular interactions (and the internal pressure probably can not explain such a difference). It seems plausible that this would be a "gas" or dark dwarf, which concentration can not increase because otherwise gravity would become repulsive at the surface.

This assumption is coherent with the observation of "quasars": the redshifts, characterized by $z = \frac{\lambda_{obs} - \lambda_0}{\lambda_0}$, result from the expansion of the universe and Einstein effect (spreading out of the durations due to the field of gravitation). The stability limit imposes Einstein effect not exceeding $z \approx 0{,}615$ for the photons emitted on the surface [6].

However, a certain number of quasars, whose distance is known by their interaction with close galaxies, seem to have a larger contribution of the Einstein effect. This can be explained more simply if the quasars consist of a "gas" of stars: some of the photons are emitted from the inside, giving a larger Einstein effect.

Knowing that the galaxies contain "black matter" [9], a possible interpretation would be that there exist many dark dwarf, being able to form a "gas". If, at the centre of some galaxies, a certain number of active stars mingle with a cluster of dark stars (possibly reactivated by some collisions), this could lead to a quasar. If, in other galaxies, a cluster without active star is formed, that could lead to a supermassive dark object resembling a "galactic black hole".

For "components" of the neutron stars type $(\mu \approx 5.10^{17}\ kg.m^{-3})$, then their average distance would be about 6000 times their radius $(R \approx 5$ km). That seems more or less reasonably plausible; moreover the light emitted by the quasars could come not from active stars, but from some collisions between the stars constituting the cluster.

With such assumptions, a phenomenon would remain to be understood: usual models based on a “big bang” predict a too small lifetime for the universe so that such a great proportion of stars are extinct (approximately six times more than active stars).

5. Cosmological models

If one imagine, to significantly larger scale, a universe containing “hyper-clusters” of matter, of specific mass $\mu \approx 10^{-27}$ kg.m^{-3}, corresponding to the stability limit, then their radius would be a few tens of billions of light years (and their mass of the order of 10^{53} kg). This is usually what is considered as the size of the universe; here one supposes these hyper-clusters included in a larger universe, containing elsewhere significantly less matter.

In the case where two such hyper-clusters enter in collision; the whole would then exceed the limit and the gravitational field would become violently repulsive between the two singular spheres where $r = a(r)$. This would correspond to an “hyper black hole”, very unstable, thus exploding as an hyper-nova.

However, on such a scale, the explosion would be very slow. How would seem the space around an observer situated in the expanding zone?

The local gravitational fields there would not be significantly disturbed by the large-scale behaviour.

Thus, the observer located in the expanding zone would receive no light either from the inner zone, or from the outer zone, each one hidden by a singularity. The light can cross the singularities in the vicinity of the local “inhomogeneousness” (“defects” of the singularities); this however requires an almost infinite apparent duration). The observer would therefore probably feel like in a limited and expanding universe.

There would be possibly moreover one difficulty for geometrical interpretation since the property $\frac{dr}{d\rho} < 0$ characterizing this zone could give the impression of seeing “outside” what corresponds to the “inside” of the hyper-cluster (and conversely). The anisotropy would it be obvious? Such configurations would probably benefit from being studied in more detail.

Several models have been proposed, involving a “dark energy” acting in some cases as a “repulsive gravity”, it seems in fact that may be there is not even need to look for such artifices [5, 7, 8].

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